# Dilute Resonating gases and the third virial coefficient [1]

Paulo F. Bedaque and Gautam Rupak

We study dilute gases with short range interactions and large two-body scattering lengths. At temperatures between the condensation temperature and the scale set by the range of the potential there is a high degree of universality. The first two terms in the expansion of thermodynamic functions in powers of the fugacity $z$, which measures the diluteness of the system, are determined by the scattering length only. The term proportional to $z^3$ depends only on one new parameter describing the three-body physics. We compute the third term of the expansion and show that, for many values of this new parameter, the $z^3$ term may be the dominant one.

Typically, the size of the two-body scattering length is comparable to the range of the force, $a \approx R$. However, there are important situations where the interactions are fine-tuned in such a way as to make $a \gg R$, even though other low-energy parameters like the effective range $r_0$, etc., still have the size expected on dimensional grounds $r_0 \approx R$. We have in mind two of these situations. The first one, a gas of neutral atoms, has received enormous experimental and theoretical attention recently. The range of the interactions between the atoms is set by the length scale $R \sim (MC_6/\hbar^2)^{1/4}$ of the van der Wals force $-C_6/r^6$. In some atomic species like $^{87}$Rb and $^4$He the fine tuning needed for large values of $a$ is provided by Nature. In the case of $^4$He atoms for instance, $a = 104\mathring{A}$, a value much larger than $R \sim 5\mathring{A}$ or the effective range $r_0 \approx 7\mathring{A}$. More importantly, an external magnetic field can be applied in order to artificially modify the scattering length, making it a *tunable* parameter (Feshbach resonance) . The second example of unnaturally large scattering lengths is a gas of neutrons (and protons, if the Coulomb force can be disregarded). The range of the nuclear forces is of the order of the Compton wavelength of the pion ($R \sim \hbar/m_\pi \sim 1.5fm$), but the scattering lengths are significantly larger (5.42 fm for the proton-neutron in the spin-triplet state to $-18.8$ fm for neutron-neutron in the spin-singlet state).

The typical momentum of the particles in a gas is set by the larger of the inverse interparticle distance $n^{1/3}$ and the inverse thermal wavelength $\lambda\!\!\!^{-} = (MT/2\pi)^{-1/2}$ (from now on we will use $\hbar = 1$). In the natural case, $a \approx R$, the universal regime occurs for $1/\lambda\!\!\!^{-}, n^{1/3} \lesssim 1/a \approx 1/R$. This regime is essentially perturbative in the sense that, at any given order of the expansion in powers of $1/R$, a properly set up diagrammatic expansion involves only a finite number of diagrams. On the other hand when $1/\lambda\!\!\!^{-}, n^{1/3} \approx 1/R$ the details of the particle interactions are important and each system should be studied in a case by case basis. The presence of large scattering lengths opens up an intermediate regime, where the typical wavelength $\lambda$ of the particles is comparable to $a$, but still smaller than $R$ . The problem is no longer perturbative, as evidenced by the fact that bound states of size $\approx a$ are expected, but some degree of universality should still hold. This regime can be attained at very low temperatures and a moderately high densities $na^3 \sim 1 \gtrsim nR^3$ or at a moderately high temperatures $a/\lambda\!\!\!^{-} \sim 1 \gtrsim R/\lambda\!\!\!^{-}$ and a very low densities.

The non-trivial part of the third virial coefficient is given by the diagram in Fig. **??**. Part of the computation is done numerically, and the result is a *universal* function of the scattering length and the three-body force.

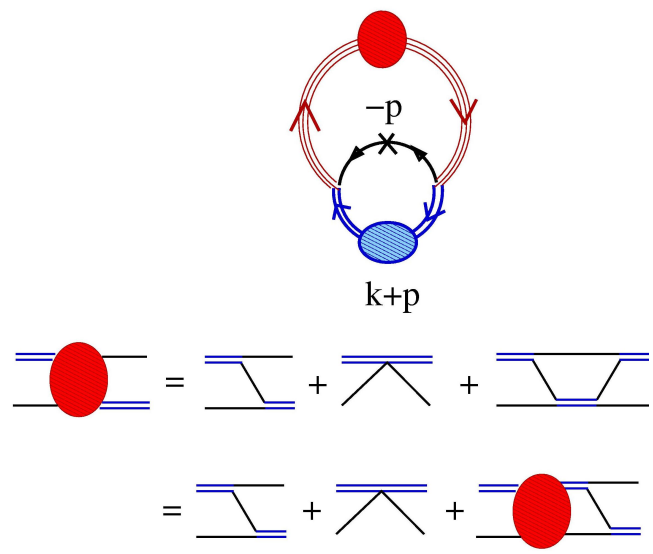


FIG. 1: Graph determining the third virial coefficient close to a Feshbach resoannce.

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