Hagedorn Thermostat: A Novel View of Hadronic Thermodynamics

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The statistical bootstrap model (SBM) [1] gave the first evidence that an exponentially growing hadronic mass spectrum $g_H(m) \to \exp[m/T_H]$ for $m \to \infty$ could lead to new thermodynamics above the Hagedorn temperature T_H. For almost four decades its grand canonical formulation was used in a variety of applications related to hadroproduction. Recently, using the microcanonical formulation, we showed [2] that in the absence of any restrictions on m, resonances with the Hagedorn mass spectrum behave as a perfect thermostat of constant temperature T_H and perfect chemical reservoir, i.e. they impart the Hagedorn temperature T_H to particles which are in thermal contact and force them to be in chemical equilibrium.

Therefore, the entire framework of the SBM which is based on the grand canonical ensemble must be revisited. In fact, it is necessary to return to the foundations of the statistical mechanics of hadrons and study the role of the Hagedorn mass spectrum for finite masses of hadronic resonances above the cut-off value m_o, below which the hadron mass spectrum is discrete. Such an analysis for an arbitrary value of the power prefactor in $g_H(m)$ (see below) is important for a better understanding the experimental data on elementary particle collisions.

Consider the microcanonical ensemble of N_B Boltzmann point-like particles of mass m_B and degeneracy g_B, and N_H hadronic point-like resonances of mass m_H with a mass spectrum $g_H(m_H) = \exp[m_H/T_H](m_o/m_H)^a$ for $m_H \geq m_o$ which obeys the inequalities $m_o \gg T_H$ and $m_o > m_B$. Then the microcanonical partition in nonrelativistic case reads as $(N_H + N_B \gg 1)$

$$\Omega_{nr} = \left[V g_H(m_H)\, [2m_H]^{\frac{3}{2}}\, I_{\frac{1}{2}} \right]^{N_H} / \; N_H! \; / \; N_B!$$
$$\left[V g_B\, [2m_B]^{\frac{3}{2}} \;\; I_{\frac{1}{2}} \right]^{N_B} E_{kin}^{\frac{3}{2}(N_H+N_B)} / \left[\tfrac{3}{2}(N_H+N_B) \right]! \,, \quad (1)$$

where $E_{kin} = U - m_H N_H - m_B N_B$ is the kinetic energy of the system, and the auxiliary integral is denoted as $I_b \equiv \int\limits_0^\infty \frac{d\xi}{(2\pi)^2}\, \xi^b\, e^{-\xi}$.

For a single Hagedorn thermostat, $N_H = 1$, we treat the mass of Hagedorn thermostat m_H as a free parameter and determine the value which maximizes the entropy of the system. We showed that the solution $m_H^* > 0$ of

$$\frac{\delta \ln \Omega_{nr}}{\delta\, m_H} = \frac{1}{T_H} + \left(\frac{3}{2} - a \right) \frac{1}{m_H^*} - \frac{3(N_B+1)}{2\, E_{kin}} = 0 \quad (2)$$

provides the maximum of the system's entropy and defines the temperature of the system

$$T^*(m_H^*) \equiv \frac{2\, E_{kin}}{3(N_B+1)} = \frac{T_H}{1 + \left(\frac{3}{2} - a \right) \frac{T_H}{m_H^*}} \,. \quad (3)$$

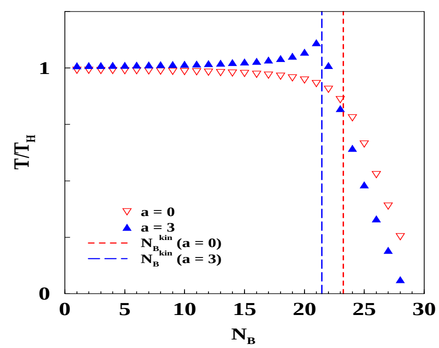

FIG. 1: A typical behavior of the system's temperature as the function of the number of Boltzmann particles N_B for $a = 3$ and $a = 0$ for the same value of the total energy $U = 30\, m_B$. Due to the thermostatic properties of a Hagedorn resonance the system's temperature is nearly constant up to the kinematically allowed value N_B^{kin} [3].

Thus, as $m_H^* \to \infty$ it follows that $T^*(m_H^*) \to T_H$, while for finite $m_H^* \gg T_H$ and $a > \frac{3}{2}$ ($a < \frac{3}{2}$) the temperature of the system in equation (3) may differ from T_H, but, as shown in Fig. 1, the maximal deviation is about 10 - 20 %, as long as there is sufficient energy in the system to keep m_H^* above the lower mass cut-off m_o. Also Fig. 1 shows that for $a > 3/2$ the Hagedorn temperature is *not a limiting temperature,* as it is commonly believed in the grand canonical SBM.

The analysis shows that fragmentation of a single heavy Hagedorn thermostat into several does not change the temperature of the system. This is so because the exponential part of the mass spectrum $g_H(m_H)$ is indifferent to splitting the mass of a single Hagedorn thermostat m_H into masses m_H^i of several thermostats, since the energy conservation requires $m_H = \sum_i m_H^i$, whereas the prefactor $(m_o/m_H^i)^a$ does not affect much the corresponding exponent $\exp[m_H^i/T_H]$. This finding not only explains why the observed inverse slopes (temperatures) of hadrons produced in high energy elementary particle collisions are 175 ± 15 MeV, but it also justifies the main assumption of the canonical formulation of the statistical hadronization model [4] that smaller clusters may be reduced to a single large cluster.

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