Statistical Properties of the Linear Sigma Model *

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The statistical equilibrium properties of the linear sigma model are studied, with a view towards characterizing the field configurations employed as initial conditions for numerical simulations of the formation of disoriented chiral condensates in high-energy nuclear collisions.

The $O(4)$ chiral field $\phi = (\sigma, \boldsymbol{\pi})$ is subject to a non-linear interaction, $V = (\lambda/4)(\phi^{\,2} - v^2)^2 - H\sigma$. The field can be decomposed into its spatial average, the order parameter, and the fluctuations, the quasi-particles, and enclosed in a box.

The quantized quasi-particle modes are described approximately by Klein-Gordon dispersion relations with an effective mass that depends on both the temperature and the magnitude of the order parameter:

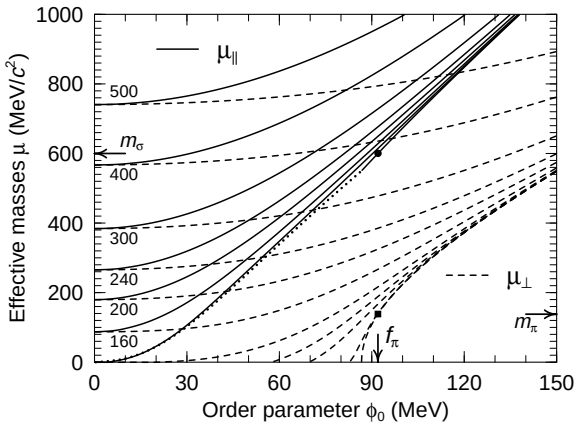

Figure 1: The effective masses $\mu_{\parallel}$ and $\mu_{\perp}$ as functions of ϕ_0 for a range of temperatures, calculated in the thermodynamic limit, $L\to\infty$. For temperatures above T_c, the curves start at $\phi_0 = 0$ with degenerate values, whereas below T_c they only exist if ϕ_0 is sufficiently large. The corresponding starting points for $\mu_{\parallel}$ are connected by the dotted curve (only T=0 is shown). Also shown are the free masses m_σ and m_π.

* LBNL-38125 (1996); Physical Review D (in press).

The thermal fluctuations are instrumental in shaping the effective potential for the order parameter. As the system is cooled the field fluctuations subside, causing a smooth change from the hot phase having approximate chiral symmetry towards the broken phase:

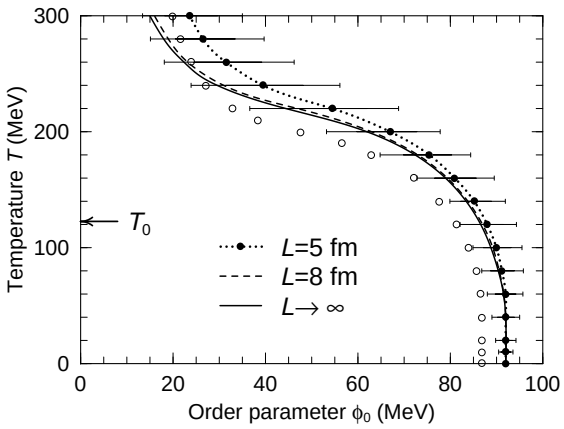

Figure 2: The magnitude of the order parameter, ϕ_0. For a box with L=5 fm, the bars show the full width at half maximum of the thermal distribution of ϕ_0 and the open dots show the centroids for the idealized case having H=0; the behavior is qualitatively similar even though the nature of the phase transition changes.

Of practical interest is the fact that the equilibrium field configurations can be sampled in a simple manner, thus providing a convenient means for specifying the initial conditions in dynamical simulations of the non-equilibrium relaxation of the chiral field.

The corresponding correlation function is briefly considered and used to calculate the spectral strength of radiated pions.

Finally, by propagating samples of initial configurations by the exact equation of motion, it has been ascertained that the treatment is sufficiently accurate to be of practical utility.