^{39}Ca (p, gamma) ^{40}Sc reaction in a microscopic model

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The $^{39}Ca(p,g)^{40}Sc$ reaction in a microscopic model

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I. INTRODUCTION

Essentially 2 problems in nuclear astrophysics:

- very low energies (small cross sections)
- radioactive nuclei in hot processes

→ A theoretical support is necessary

3 categories of theoretical models :

a) « Ab initio » models
The cross sections are determined from the wave functions of the system.
Examples: potential model
DWBA
Microscopic cluster models

b) Fitting procedures (R-matrix, K-matrix)
Experimental cross sections are fitted and extrapolated down to astrophysical energies
(**→ experimental data are necessary**)

c) Statistical models
Must be applied for high-level densities (heavy nuclei)

II. THE MICROSCOPIC CLUSTER MODEL

II.1 Definitions

- Hamiltonian

$$H = \sum_{i}^{A} T_i + \sum_{i,j}^{A} V_{ij}$$

with T_i = kinetic energy of nucleon i
V_{ij} = interaction between nucleons i and j (almost *independent of the system*)

Not solvable as soon as A> 3

- Cluster approximation

The nucleons are assumed to be grouped in clusters

$$\Psi_{lm} = \mathcal{A} \; \Phi_1 \Phi_2 \, g_l(r) \, Y_l^m(\hat{r}) \quad \text{(spins zero)}$$

with $\mathcal{A}$ = antisymmetrization operator
r = relative coordinate
Φ_1, Φ_2 = internal wave functions **(=data)**
$g_l(r)$ = relative function **(=output)** determined from the Hamiltonian

Generalization for multichannel systems (spins $\neq 0$)

$$\psi^{JM} = \mathcal{A} \sum_c \left[\left[\phi_c^{I_1} \otimes \phi_c^{I_2} \right]^{I} \otimes Y_l(\hat{r}) \right]^{JM} g_{cl}^{JM}(r)$$

with c = channel

II.2 Inputs

1. V_{ij} : the nucleon-nucleon interaction

   (effective force, adapted to gaussian wave functions)

2. ϕ_1, ϕ_2 : internal wave functions

   - Shell-model wave functions : α, ^{16}O, ^{39}Ca
   - Multi cluster : ex : $\begin{cases} {}^{7}Be = \alpha + {}^{3}He \\ {}^{12}C = \alpha + \alpha + \alpha \end{cases}$

II.3 Outputs

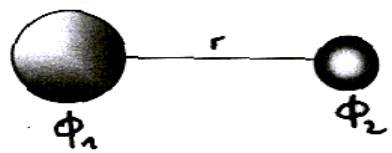
Basically : the radial function $g_\ell(r)$

→ - Spectroscopic properties (energies, widths, $B(E2)$...)
  - Scattering amplitudes
  - Cross sections

II.4 Applications

- Halo nuclei (^{6}He, ^{11}Li, ^{19}C, ^{31}Ne...)
- Astrophysics ($^{7}Be(p,\gamma)$, $^{12}C(\alpha,\gamma)$, $^{39}Ca(p,\gamma)$)

The Generator Coordinate Method (GCM)



g_l expanded in a Gaussian basis:

$$g_l(r)=\sum_n f^l(R_n)\exp(-\mu(r-R_n)^2/b^2)$$

R_n = generator coordinate
b = oscillator parameter
μ = reduced mass

$$\Psi_{lm} = \mathcal{A}\ \Phi_1\Phi_2\, g_l(r)\, Y_l^m(\hat{r})$$

$$=\sum_n f^l(R_n)\underbrace{\mathcal{A}\ \exp(-\mu(r-R_n)^2/b^2)\,\Phi_1\Phi_2\, Y_l(\hat{r})}_{\text{Slater determinant}}$$

- linear combination of Slater Determinants (well adapted to numerical calculations)
- *calculation of $g_l(r)$ replaced by $f_l(R_n)$ (= generator function)*
- problems with the asymptotic behavior (Gaussian)

$$g_l(r) \rightarrow \exp\left(-\frac{\mu r^2}{b^2}\right) \quad : \text{ not physical}$$

The microscopic R-matrix method

- Adaptation of the standard R-matrix theory (Lane and Thomas, Barker…)
- 2 regions:

  internal (Slater determinants)

  $$r < a \; : \; \psi^{lm}_{int} = \mathcal{A}\, \phi_1 \phi_2\, Y^m_l(\hat{r}) \sum_n f^l(R_n) \exp\left(-\frac{(r-R_n)}{b^2}\right)$$

  external (Coulomb or Witthaker functions)

  $$r \geq a \; : \; \psi^{lm}_{ext} = \mathcal{A}\, \phi_1 \phi_2\, Y^m_l(\hat{r}) \left[I_l(r) - U_l\, O_l(r) \right] \quad E>$$

  $$= \mathcal{A}\, \phi_1 \phi_2\, Y^m_l(\hat{r})\, W_{-\eta, l+\frac{1}{2}}(2kr) \quad E<$$

- Matching conditions provide:
  - Bound-state energies (for E<0)
  - Collision matrices (E>0)
  - Wave functions valid in the whole space

In summary:

With the GCM and MRM, we have a unified description of

- Bound states
- Resonant states
- Scattering states

➔ applications to spectroscopy (halo nuclei)
reactions (astrophysics)

Comments about the spectroscopic factors

1. Theoretical definition

Nucleus formed of 2 clusters A and B:

wave function: $\Psi^{JM} = \mathcal{A}\ \Phi_A \Phi_B Y_J^M g_J(\rho)$

overlap integral: $I(r) = \langle \Phi_A \Phi_B \delta(r-\rho) | \Psi^{JM} \rangle$

spectroscopic factor: $S_{th}^J = \int I(r) r^2 dr$ *involves the wave function over the whole space*

reduced width: $\theta^2 \div |g_J(a)|^2$ *involves the wave function at the channel radius a*

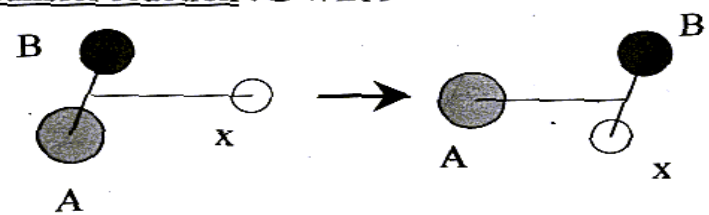
2. Experimental definitions

a) Capture reaction to bound state J : A(B,γ)C reaction

Spectroscopic factor defined by $S_c^J = \frac{\sigma_{exp}}{\sigma_{th}}$

where σ_{th} is generated by a potential A+B

b) Transfer reaction : DWBA



$\sigma_{exp} = S_t(AB) S_t(Bx) \times \sigma_{DWBA}$ *Provides the product $S_t(AB)S_t(Bx)$*

c) Resonances: $S_R^J = \frac{\Gamma_{exp}}{\Gamma_{sp}}$ *Where Γ_{sp} is the single-particle width generated by a A+B potential*

3. **Limitations**

a. Assumptions

1) $S_{th}=S_c=S_t=S_R$ validity???

2) S is charge independent (use of ^{40}K spect. factors for ^{40}Sc)

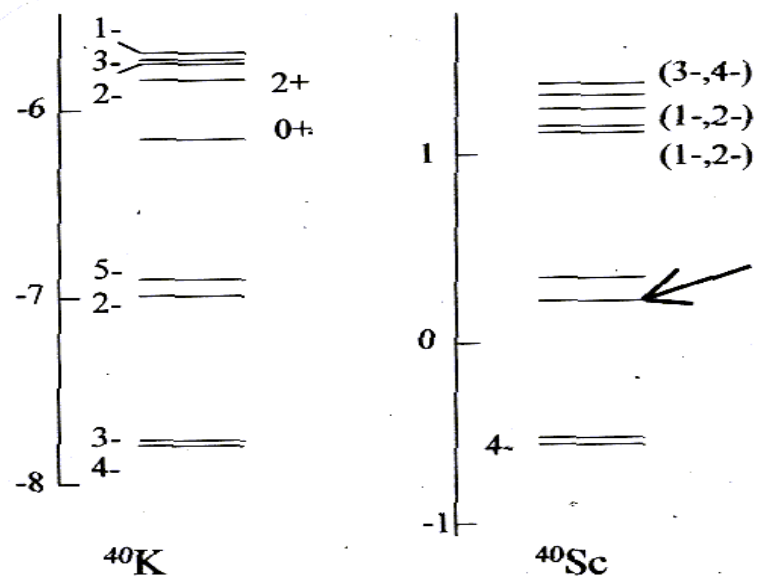
b. Model validity
If S is very different from 1 ⇨ what about the potential?

c. Resonances
The potential model can not account for several nearby resonances in the same partial wave

The $^{39}Ca(p,\gamma)^{40}Sc$ reaction

- *Important in the rp process*: determines the nucleosynthesis of elements with $A \geq 40$ ($T > 3.10^8$ K)

- **Experimental data:**
  - ^{40}Sc, ^{40}K *spectra*
  - electromagnetic transitions in ^{40}K
  - Spectroscopic factors in ^{40}K



- **Calculation:** *Iliadis et al. (1999)*
  - Non-resonant: potential model
  - Resonant: information in ^{40}K

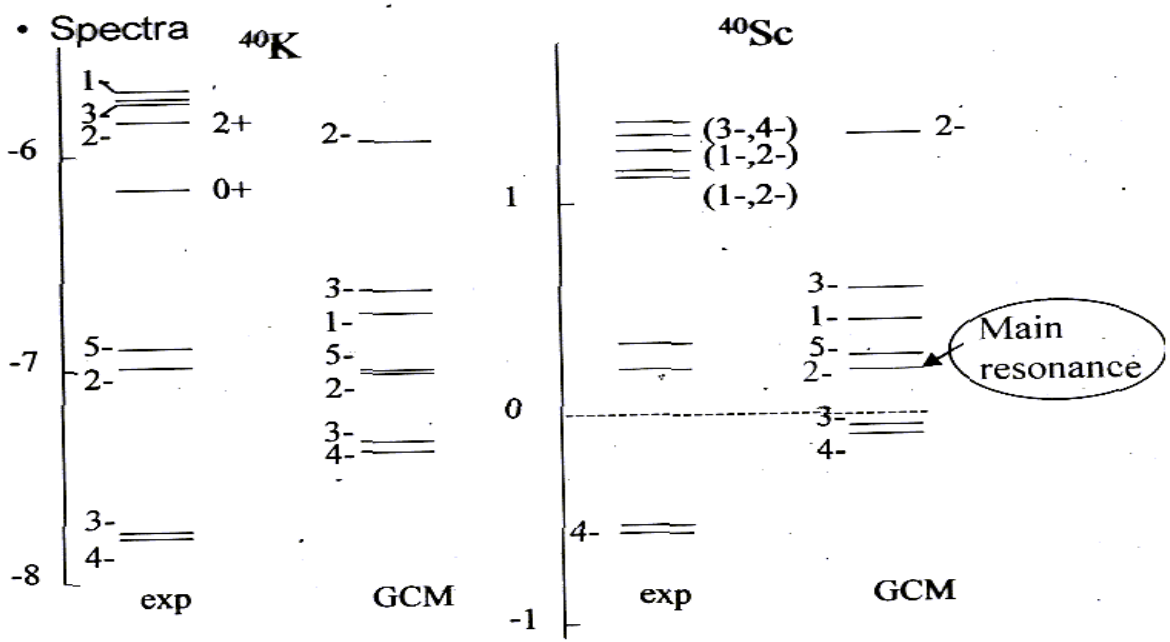
Present calculation:

- Wave functions : $^{39}Ca(3/2^+,1/2^+,5/2^+) + p$
  with ^{39}Ca= sd-shell wave function with one neutron hole
  → well adapetd to a microscopic calculation

- Nucleon-nucleon interaction : **Volkov force**
  (one parameter: $M \approx 0.6$)

  here: M is determined on the energy of the first resonance in
  $^{39}Ca(p,\gamma)^{40}Sc$

- Spectra



Spectroscopic properties

	GCM	exp
^{39}K		
μ(gs) (μ_N)	0.12	0.39
Q(gs) (e.fm^2)	4.5	4.9 ± 0.4
^{39}Ca		
μ(gs) (μ_N)	1.15	1.02
^{40}K		
μ(gs) (μ_N)	-1.83	-1.30
μ(3-) (μ_N)	-1.80	-1.29 ± 0.09
Q(gs) (e.fm^2)	-6.3	-6.1 ± 0.5

Transition probabilities in ^{40}K

		GCM	exp
$3^- \rightarrow 4^-$	M1	0.12	0.35
	E2	6.2	
$2^- \rightarrow 4^-$	E2	5.6	9.3
$2^- \rightarrow 3^-$	M1	0.15	0.315
	E2	13.3	
$5^- \rightarrow 4^-$	M1	0.03	0.06
	E2	15.6	11.5
$5^- \rightarrow 3^-$	E2	9.5	19.2

^{40}K and ^{40}Sc spectroscopic factors

- $^{39}K(3/2^+) + n \rightarrow$ channel spin I=1 and 2
- Experiment: *Fink and Schiffer (Nucl. Phys. A225, 1974, 93)* with the $^{39}K(d,p)^{40}K$ reaction
- Calculation: multichannel and single-channel

State		GCM				exp	
		(1,1)	(1,3)	(2,1)	(2,3)	L=1	L=3
4^-	^{40}K	-	0.36	-	0.71		0.9
	^{40}Sc	-	*0.35*	-	*0.70*		
	1 ch.	-	0.36	-	0.72		
3^-	^{40}K	-	0.46	0.37	0.24	0.02	0.9
	^{40}Sc	-	*0.46*	*0.37*	*0.25*		
	1 ch.	-	0.41	0.43	0.22		
2^-_1	^{40}K	0.45	0.03	0.60	~0	0.014	0.9
	^{40}Sc	*0.44*	*0.02*	*0.61*	*~0*		
	1 ch.	0.44	0.02	0.61	~0		
2^-_2	^{40}K	~0	0.78	0.04	0.07	0.5	
	^{40}Sc	*~0*	*0.78*	*0.07*	*0.08*		
	1 ch.	*0.57*	*0.04*	*0.45*	*0.01*		

- Charge symmetry is OK
- For the 2^-_2 state: strong influence of the number of channels
- Good agreement with experiment for 4^- and 3^- states
- *Significant differences for 2^- states*

Properties of the 2^- resonance

Iliadis et al (1999)

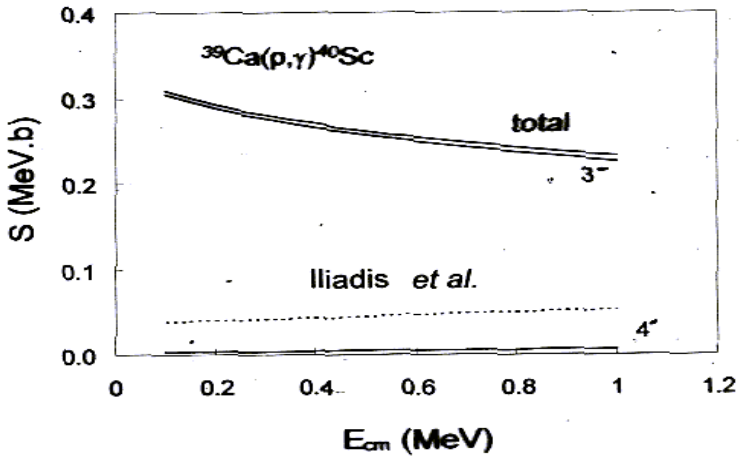
- $\Gamma_\gamma = 1.6$ meV
- $\Gamma_p = 2.0 \times 10^{-9}$ eV
  (estimated from spectroscopic factors in ^{40}K)

Present:

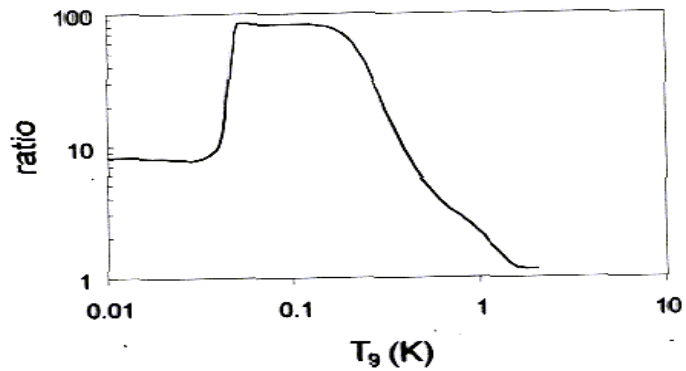
- $\Gamma_\gamma = 1.64$ meV
- $\Gamma_p = 1.2 \times 10^{-7}$ eV $\rightarrow$ strong difference

For other states: small differences

Non-resonant S-factor



Reaction rate



→Difference up to á factor of 100!

Due to the different methods

Conclusions

- Strong difference (for the main resonance) with previous calculation based on spectroscopic factors

- Spectroscopic factors: used to compensate model deficiencies (number of channels)
  → different models would give different spectroscopic factors

- Different definitions of spectroscopic factors:
  - Theory: from *overlap integrals*
  - Experiment: ratio of experimental and theoretical cross sections: transfer (here $^{39}K(d,p)^{40}K$) or capture
  - Are all definitions equivalent???

- Here:
  - No use of spectroscopic factors
  - Several tests of the model (^{40}K spectroscopy)
  - Many other reactions ($^{7}Be(p,\gamma)^{8}B$, $^{13}N(p,\gamma)^{14}O$,...)

→ Need for direct data!