REDSHIFT ACCURACY REQUIREMENTS FOR FUTURE SUPERNOVA AND NUMBER COUNT SURVEYS

Dragan Huterer[1], Alex Kim[2], Lawrence M. Krauss[1,3] and Tamara Broderick[4]

[1]Department of Physics and Center for Education and Research in Cosmology and Astrophysics, Case Western Reserve University, Cleveland, OH 44106

[2]Physical Sciences Division, Lawrence Berkeley National Laboratory, Berkeley, CA 94720

[3]Department of Astronomy, Case Western Reserve University, Cleveland, OH 44106

[4]Laurel High School, Cleveland, OH 44106

Draft version August 9, 2004

ABSTRACT

We investigate the required redshift accuracy of type Ia supernova and cluster number-count surveys in order for the redshift uncertainties not to contribute appreciably to the dark energy parameter error budget. For the SNAP supernova experiment, we find that, without the assistance of ground-based measurements, individual supernova redshifts would need to be determined to about 0.002 or better, which is a challenging but feasible requirement for a low-resolution spectrograph. However, we find that accurate redshifts for $z < 0.1$ supernovae, obtained with ground-based experiments, are sufficient to immunize the results against even relatively large redshift errors at high z. For the future cluster number-count surveys such as the South Pole Telescope, Planck or DUET, we find that the purely statistical error in photometric redshift is less important, and that the irreducible, systematic bias in redshift drives the requirements. The redshift bias will have to be kept below 0.001-0.005 per redshift bin (which is determined by the filter set), depending on the sky coverage and details of the definition of the minimal mass of the survey. Furthermore, we find that X-ray surveys have a more stringent required redshift accuracy than Sunyaev-Zeldovich (SZ) effect surveys since they use a shorter lever arm in redshift; conversely, SZ surveys benefit from their high redshift reach only so long as some redshift information is available for distant ($z \gtrsim 1$) clusters.

Subject headings: cosmology: theory – large-scale structure of universe

1. INTRODUCTION

Two of the most promising methods to measure cosmological parameters, in particular those describing dark energy, are distance measurements of type Ia supernovae (SNe Ia) and number counts of clusters of galaxies in the universe. SNe Ia have provided original direct evidence for dark energy (Riess et al. 1998, Perlmutter et al. 1999) (for earlier, indirect evidence, see Krauss and Turner 1995 or Ostriker and Steinhardt 1995) and are currently the strongest direct probe of the expansion history of the universe (Tonry et al. 2003, Knop et al. 2003). Their principal strength is the simplicity of relating the observable – which is essentially the luminosity distance – to cosmological parameters, and also the fact that each supernova redshift-magnitude pair provides a distinct measurement of a combination of those parameters. Number-counts, on the other hand, use the fact that galaxy clusters are the largest collapsed structures in the universe that have undergone a relatively small amount of post-processing. Their distribution in redshift can be reliably calculated in a given cosmological model. The evolution of cluster abundance is principally sensitive to the comoving volume and growth of density perturbations (Haiman, Mohr & Holder 2001) and this cosmological probe is expected to reach its full potential with upcoming and future wide-field surveys.

Rapid improvement in the accuracy of measuring cosmological parameters implies that various systematic uncertainties, previously ignored, now have to be controlled and understood quantitatively. In the case of supernova measurements, an example is provided by the proposed SuperNova/Acceleration Probe (SNAP) satellite (Akerlof et al. 2004) whose goals for measuring the equation of state of dark energy w and its variation with redshift dw/dz drive the requirements on the systematic control that are considerably more stringent than those attainable with current surveys. Similarly, the principal systematic difficulty in cluster counts is in establishing the relation between observable quantities (X-ray temperature or Sunyaev-Zeldovich flux decrement), and the cluster's mass which is necessary for comparison with theory. The mass-temperature relation, for example, is known to have a considerable scatter and is currently poorly determined, with fairly large intrinsic statistical errors and considerable systematic disagreements between different authors (see e.g. Fig. 2 in Huterer & White 2002). The cleanest way to include the mass-observable relation might be to determine it from the survey itself (this is known as "self-calibration"; Levine, Schulz & White 2002, Majumdar & Mohr 2003, Hu 2003, Lima & Hu 2004), but this will almost certainly lead to degradations in parameter accuracies. Future surveys will require a careful accounting of all systematics – theoretical and observational.

In this paper we concentrate on one of the most basic ingredients of supernova and cluster count measurements: the determination of redshift. In the case of SNe Ia spectroscopic observations are necessary to identify the supernova type, and redshift is then supplied for free. Recently completed and ongoing surveys have sufficiently poor magnitude uncertainty and relatively low statistics and relatively weak control on known systematics, so that the spectroscopic redshift error is small enough for the redshifts to be considered perfectly known. However, as we shall see,

future supernova observations require such accurate redshifts that even the spectroscopic accuracy is not *a priori* guaranteed to be sufficient.

In the cluster count case, the situation is even more interesting, as spectroscopic observations will not be possible for all clusters, which may number in the tens of thousands. One will therefore rely on photometric redshifts. Although photometric redshifts are already impressively accurate (e.g. Fernández-Soto et al. 2002, Csabai et al. 2003, Collister & Lahav 2003, Vanzella et al. 2003), we shall find that their bias (the difference between the mean photometric value and the true value at any redshift) needs to be kept exceedingly small for the redshift error not to contribute appreciably to the total error budget. Our analysis is timely, as follow-up surveys to obtain cluster redshifts, such as that at Cerro-Tololo International Observatory, are about to get underway soon. Our analysis also complements recent analysis of the effect of systematic errors on future SN Ia measurements (Kim et al. 2003, Frieman et al. 2003) and a variety of related analyses regarding the cluster number-count surveys (e.g. Bartlett 2000, Holder & Carlstrom 2001, White, Hernquist & Springel 2002, White, van Waerbeke & Mackey 2002, Benson, Reichardt & Kamionkowski 2002, White 2003).

The paper is organized as follows. In section 2 we outline the procedure to include the redshift uncertainty in the standard Fisher-matrix parameter estimation. In Sec. 3 we discuss the redshift requirements for future supernova surveys, while in Sec. 4 we do the same for future cluster count surveys. We conclude in Sec. 5. Our fiducial model is a flat universe with matter energy density relative to critical of $\Omega_M = 0.3$ and the equation of state of dark energy $w = -1$. Other cosmological parameters, necessary for the cluster abundance calculation, are discussed in Sec. 4.

2. METHODOLOGY

Let us assume we have an observable $O(z)$ from which we want to determine P cosmological parameters $\theta_1, \ldots, \theta_P$. Since the number of observed objects is large (thousands for future SNe Ia surveys and of order ten thousand for cluster surveys), we bin observations in Q redshift bins centered at z_k $(1 \leq k \leq Q)$ each with width Δz_k.

To include the redshift uncertainty, we treat the bin centers $z_1, \ldots, z_Q$ as Q additional nuisance parameters $\theta_{P+1}, \ldots, \theta_{P+Q}$. Variation of any of these parameters moves the location of the whole corresponding redshift bin. We use the Fisher matrix formalism in order to estimate all $P + Q$ parameters simultaneously, and we give priors to redshift parameters that represent how accurately they are independently determined.

The $(P + Q) \times (P + Q)$ Fisher matrix is given by

$$F_{ij} = \sum_{k=1}^{Q} \frac{N_k}{\sigma_O(z_k)^2} \frac{\partial O(z_k)}{\partial \theta_i} \frac{\partial O(z_k)}{\partial \theta_j} \qquad (1)$$

where N_k is the number of objects in the k-th bin. The observable $O(z_k)$ is the mean apparent magnitude of a supernova $m(z_k)$ in a given redshift bin, or else the number of clusters $N(z_k)$ in a bin. Note that the redshift parameter representing the ith bin affects the observable $O(z_k)$ only if $i = k$. Therefore

$$\frac{\partial O(z_k)}{\partial \theta_{P+i}} = \frac{\partial O(z_i)}{\partial \theta_{P+i}} \delta_{ik} \quad \text{for} \quad i \in [1, \ldots, Q] \qquad (2)$$

and the expression for the Fisher matrix simplifies accordingly for the redshift parameter terms.

We assume the error in the observable $O(z_k)$ is Gaussian-distributed with standard deviation $\sigma_O(z_k)$. With that assumption, any Gaussian prior imposed on the individual object, $\sigma_{\rm prior}$ will be equivalent to imposing a prior of $\sigma_{\rm prior}/\sqrt{N_k}$ on the observable representing the k-th redshift bin since there are N_k objects in this bin. Note that the number of redshift bins, Q, needs to be large enough to retain the shape information of the function $O(z)$; we use steps of 0.02 for both SNe Ia and number counts, and have checked that a higher number of bins leads to negligible changes in all of our results. Note too that these bins are used for computational accuracy; they should not be confused with the *physical* redshift bins which are determined by the filter set of the experiment and which we later discuss. Finally, we ignore the cosmic variance contribution to the error in number-count surveys, since it has been shown that cosmic variance becomes small for the high-redshift cluster surveys with relatively high mass threshold (Hu & Kravtsov 2003), which is the case we study in this paper.

While our Fisher matrix formalism assumes the redshift errors to be Gaussian, it is conceivable that the errors will have significant non-Gaussian tails and/or pronounced skewness. This may especially be true for the photometric redshift errors, where a small fraction of redshifts may have a large deviation from their true value. We, however, do not expect that this effect will significantly affect our results: as we discuss in Sec. 4, the results are mainly sensitive to the systematic bias in redshift rather than the width of the redshift distribution. This bias requirement can then be compared to the expected observational bias derived from the actual photometric redshift distribution – whether it is Gaussian or not – to guide us in judging whether a particular photometric-redshift technique is acceptable. Detailed modeling of the photometric redshift error, taking into account various types of galaxies and their properties at any given redshift, will require a Monte Carlo approach that is outside the scope of this work.

3. TYPE IA SUPERNOVAE

3.1. *Redshift dependence of supernova measurements*

The measurement of the cosmological parameters using calibrated candles requires both the magnitude and redshift of the object in question. In supernova studies the redshift measurement is typically taken from the spectrum of the host galaxy, either from sharp emission lines or from the 4000 Å break. Up to the present, the magnitude error of high-redshift supernovae have dwarfed the redshift measurement error. As we enter an era of high-precision supernova cosmology, with significant improvement in statistical and systematic uncertainties, we need to explore the effects of redshift measurement error on the determination of cosmological parameters.

Cosmological observations of high-redshift supernovae are generally made in observer X and Y bands which roughly correspond to supernova-frame B and V bands.

Observed Type Ia supernova magnitudes are modeled as

$$m_X = M_B - \alpha(s-1) + A_B(s,z) + K_{BX}(s,z) + \mu(z;\theta_i). \quad (3)$$

The peak absolute magnitude of a supernova $M_B - \alpha(s-1)$ is a function of the "stretch" s of its light-curve shape; supernovae with higher stretch are intrinsically brighter (Perlmutter et al. 1997). The K-correction K_{BX}, a function of redshift and stretch, accounts for differences in the spectral energy distribution (SED) transmitted through the B and X bands for low- and high-redshift supernovae respectively. The extinction from the host galaxy is given by $A_B = R_B E(B-V)$ where the color excess is

$$E(B-V) = [(m_X - K_{BX}(s,z)) - (m_Y - K_{VY}(s,z))] - (B-V)_{\rm expected}(s) \quad (4)$$

where $(B-V)_{\rm expected}(s)$ is the expected supernova color, which is a function of stretch. The distance modulus μ is a function of redshift and the cosmological parameters. Gravitational lensing magnification is not considered here since its effect on the inferred magnitudes of distant supernovae is not sensitive to small variations in redshift.

The effect of redshift error on the estimated distance modulus is straightforward: a positive redshift error, dz, incorrectly gives an inflated distance to the supernova. In addition, the measurement of stretch is itself dependent on z; the stretch is obtained from the observed width W of a light curve using the formula $s = W/(1+z)$ so that an error in z propagates as $ds = -s\,dz/(1+z)$. An overestimate of redshift gives an underestimate of the stretch factor and therefore an overestimate of the expected supernova magnitude.

The extinction terms and K-correction depend on both redshift and stretch. Expected observed colors with a fixed pair of filters near the restframe B and V are bluer for slightly higher redshift supernovae. An overestimated redshift will thus give an overestimated extinction determination. In contrast, the simultaneously underestimated stretch determination overestimates the intrinsic redness of the supernova, underestimating the extinction. Additionally, stretch-dependent SED's and redshift errors introduce K-correction errors whose behavior depends on the specific redshift and filters involved.

We propagate redshift errors into errors in the expected observed peak magnitude for a canonical supernova search. We adopt a filter-set consisting of redshifted B filters such that the n-th filter has throughput $F_n(\lambda) = B(\lambda/1.16^n)$, where λ is wavelength. We adopt the empirically derived $\alpha = 1.9$ stretch–magnitude relation found by Perlmutter et al. (1997). The stretch-dependent supernova color is given as $B - V = -0.19(s-1) - 0.05$. We use the K-correction methodology given in Nugent, Kim, & Perlmutter (2002); the observer filters with effective wavelengths closest to $4400(1+z)$ and $5500(1+z)$ are associated with rest-frame B and V respectively. The host-galaxy extinction is assumed to obey the standard $R_B = 4.1$ dust model of Cardelli, Clayton, & Mathis (1989).

Figure 1 shows individual contributions to the derivative of magnitude with respect to redshift, dm/dz, as well as their sum for an $s = 1$ supernova. The K-correction and extinction errors are discontinuous and periodic in redshift as different observer filters are traversed. Note that the use of dm/dz to assess the effect of redshift errors is only approximate since the K-corrections can be highly nonlinear; however, the approximation is good for the span of redshift errors that we consider.

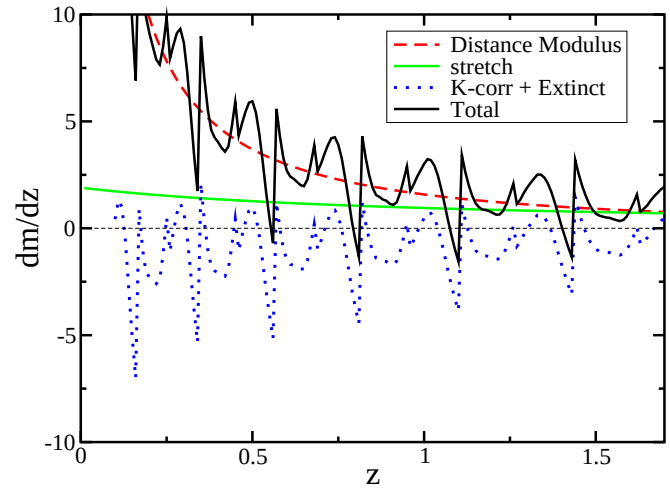

FIG. 1.— The differential effect on the expected supernova peak magnitude with variation in redshift, or dm/dz, as a function of redshift. Also shown are the contribution from individual terms; the distance modulus, stretch, K-correction, and extinction.

3.2. *Results*

We explore the effects of redshift error in the measurement of dark energy parameters based on a supernova search such as the proposed SNAP space telescope (Aldering et al. 2004). As we discuss below, more powerful experiments require more stringent control of redshift errors; therefore, our requirements for SNAP will be more than sufficient for other ground and space-based surveys in the next 5-10 years. We assume a supernova distribution with around 2700 SNe distributed between $z = 0.1$ and 1.7 together with 300 additional low-z ($z \approx 0.05$) SNe from the ground-based SN Factory (Aldering et al. 2002). The fiducial magnitude error per supernova, the quadratic sum of measurement error and intrinsic supernova magnitude dispersion, is 0.15 magnitudes. (An analysis of the effect of redshift-dependent magnitude uncertainties will be discussed in Krauss et al., in preparation.) We consider the degradation, due to imperfectly known redshifts, of the accuracy in the equation of state of dark energy σ_w, where w is assumed constant. The uncertainty σ_w is computed by marginalizing over the matter density Ω_M, the overall offset in the magnitude-redshift diagram, $\mathcal{M}$, and the redshift parameters z_i ($i \in [1,\ldots,Q]$). We also consider the degradation in the accuracy in measuring the redshift evolution in the equation of state, dw/dz, where $w(z) = w_0 + z\,dw/dz$; in this case we further marginalize over w_0 and add a Gaussian prior of 0.01 to Ω_M, to allow comparisons with other analyses in which this procedure has become standard.

Figure 2 shows the degradation in σ_w (top panel) and dw/dz (bottom panel) as a function of the redshift error per supernova. The solid line shows the case when the redshift error is constant for all SNe, whereas the dashed line represents an uncertainty growing as $dz \propto (1+z)$.

The two cases are qualitatively similar, and show that, for example, a redshift error of 0.005 per SN leads to a 25% increase in σ_w and a 7% increase in $\sigma_{dw/dz}$.

However, these results assume that the error in redshift, absolute or fractional, is the same at all redshifts. In reality, the SN Factory spectra will have a fixed high resolution. The flatter pairs of lines in Fig. 2 assume that redshifts at $z < 0.1$ have a fixed accuracy of 0.001 per SN, while those at $z > 0.1$ have the accuracy shown. The degradation in w or dw/dz is now smaller than about 10%, even for errors of 0.02 for SNe at $z > 0.1$! Therefore, accurate redshifts of low-z supernovae immunize against larger errors at higher redshifts. This conclusion is easy to understand: fixed error in redshift roughly corresponds to fixed error in distance to a supernova, while the total SN magnitude error increases linearly with distance. Therefore, the redshift error contributes a larger percent of the total error budget for low-redshift supernovae. Furthermore, low-z supernovae are crucial for parameter determination and their omission (or inclusion, but with large redshift error) would lead to nearly a factor of two degradation in the constraints on w and dw/dz (e.g. Huterer & Turner 2001). Therefore, we conclude that, provided that redshifts of low-z supernovae are measured with high accuracy, measurements of w and dw/dz are weakly sensitive to the redshift errors of high-z SNe.

For convenient reference, we associate redshift errors with the magnitude error that gives an equivalent uncertainty in w; see Fig. 3. For example, a 0.005 redshift error introduces an uncertainty equivalent to the additional 0.1 intrinsic magnitude dispersion per SN. As before, the flatter pair of lines in the same Figure shows the effect of accurate redshifts for $z < 0.1$ SNe, in which case the overall redshift uncertainty contributes little ($\lesssim 0.02$ mag in the range of redshift errors shown) to the total error budget.

The spectroscopic follow-up of high-redshift supernovae (or more appropriately, their host galaxies) from surveys must be carefully considered. Ground-based spectroscopy associated with current high-z supernova searches have more than sufficient resolution to measure redshifts to $R \equiv \lambda/\delta\lambda = 200$. (Subpixel interpolation gives a wavelength resolution several times better than the instrumental resolution R.) Very wide-field supernova searches that discover thousands or more supernovae may depend on photometric redshifts as an alternative to spectroscopic followup. Photometric redshift determination is currently limited by a statistical accuracy floor of a few percent (see Sec. 4); in this case the effect of redshift error can be comparable in size to the intrinsic corrected-magnitude dispersion of SNe Ia of $\sim$0.1 mag, although we have just shown that accurate redshifts at $z < 0.1$ will largely immunize against the overall redshift contribution to the error budget. Of course, the two more serious problems are identification of SNe Ia and control of systematics, both of which are very difficult without their spectra.

In the case of a 2-m space telescope observing $z \sim 1.7$ supernovae (such as SNAP), the Poisson noise from the zodiacal background and source can be low. Considering the noise properties of HgCdTe detectors, signal-to-noise arguments push for a low-resolution spectrograph to avoid a detector-noise-limited instrument. If the on-board spectrograph is to provide supernova redshifts, the competing needs for low and high resolution must be considered in the design.

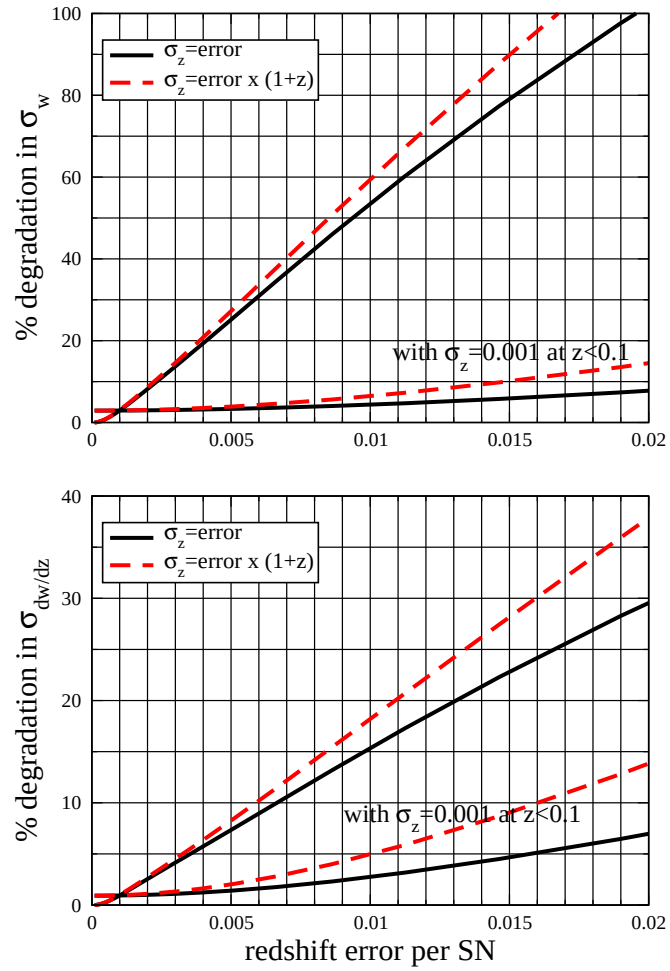

FIG. 2.— Degradation in the measurement accuracy of the equation of state w (top panel) and, alternatively, its rate of change with redshift dw/dz (bottom panel) as a function of redshift error per supernova. The solid line shows the case when the redshift error is constant for all SNe, while the dashed line shows the case when the error (plotted on the abscissa) is multiplied by (1+z). The flatter pair of curves in each panel corresponds to the case when $z < 0.1$ SNe were assumed to have a fixed redshift error of 0.001 per SN.

We end with a couple of comments. First, we do not consider the peculiar velocities since they have negligible effect on high-redshift supernovae. Second, our supernova calculations involve statistical magnitude errors only. Adding the irreducible systematic error in each redshift bin, as done in Frieman et al. (2003) for example, is straightforward and leads to a slight weakening of the required redshift accuracy. This is easy to understand, since the fiducial parameter accuracy, such as σ_w, slightly weakens in the presence of systematics, and the redshifts do not need to be known quite as accurately as in the perfect world without systematics.

4. CLUSTER NUMBER-COUNT SURVEYS

4.1. *Fiducial surveys and assumptions*

Clusters can be found using their X-ray flux; through their Sunyaev-Zeldovich temperature decrement, or through deflection of light from background galaxies due to weak gravitational lensing by the cluster. Current or upcom-

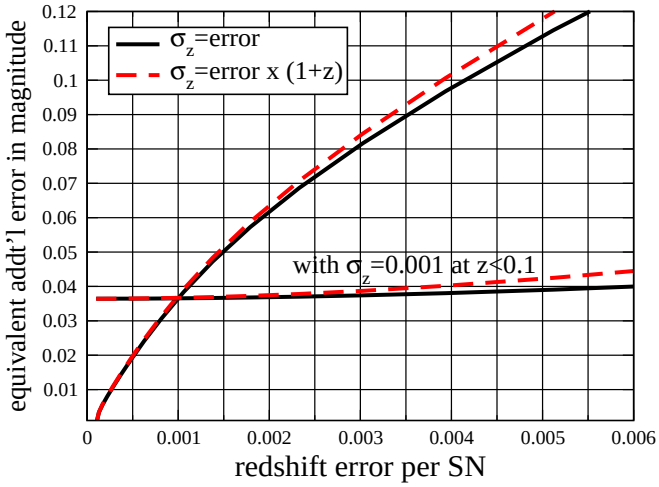

FIG. 3.— Redshift error increases the degradation in σ_w as some equivalent *additional* magnitude error, which is shown on the ordinate. The solid line shows the case when the redshift error is constant for all SNe, while the dashed line shows the case when the error (plotted on the abscissa) is multiplied by $(1+z)$. As in Fig. 2, the flatter pair of curves corresponds to the case when $z < 0.1$ SNe were assumed to have a fixed redshift error of 0.001 per SN.

ing surveys specifically suited to find clusters include XMM Serendipitous Cluster Survey (Romer et al. 2001) XMM Large Scale Structure survey (Pierre et al. 2003), ACBAR[1], Sunyaev-Zeldovich Array[2], APEX SZ survey[3] and Atacama Cosmology Telescope[4]. Cluster cosmology will realize its full potential with future wide-field surveys, such as the South Pole Telescope[5] (SPT), the space-based mission Planck[6], the proposed space-based mission DUET, and, in the next decade, SNAP.

Cluster redshifts are required in order to use clusters as a probe of cosmology, yet the large number of clusters expected in the aforementioned upcoming surveys (thousands to tens of thousands) makes it impractical to obtain their redshifts spectroscopically. Therefore, cluster abundance studies will rely on the photometric redshifts. Fortunately, cluster photometric redshifts are currently measured with very good accuracy (e.g. Bahcall et al. 2003) chiefly because the photo-z's of individual galaxies in the cluster can be averaged, leading to statistical errors in cluster redshifts of about 0.02. However, goals for the cluster abundance surveys are set high – measuring the equation of state of dark energy w to an accuracy of 5-10% and the power spectrum normalization σ_8 to about 1% – and it is worthwhile to study the required photometric redshift accuracy in order to achieve this goal. Previous studies of the cluster abundance (Haiman, Mohr & Holder 2001, Levine, Schulz & White 2002, Battye & Weller 2003, Hu & Kravtsov 2003, Mohr & Majumdar 2003a, Molnar et al. 2003) have explored the efficacy of cluster number counts as a probe of cosmology, but all assumed perfect knowledge of redshifts.

We adopt the fiducial cosmological model which is in accordance with recent results from the WMAP experiment (Spergel et al. 2003) and the Sloan Digital Sky Survey (Tegmark et al. 2003). We assume a flat universe with matter energy density relative to critical $\Omega_M = 0.3$, dark energy equation of state $w = -1$, and power spectrum normalization $\sigma_8 = 0.9$. We use the spectral index and physical matter and baryon energy densities with mean values $n = 0.97$, $\Omega_M h^2 = 0.140$ and $\Omega_B h^2 = 0.023$ respectively. We add a fairly conservative prior of 5% to each of these parameters; we checked that our results are insensitive to this prior. The parameters to which cluster surveys are most sensitive are Ω_M, w and σ_8, and we do not give any priors to these parameters. The fiducial surveys we consider determine Ω_M and σ_8 to accuracy of about 0.01 and w to about 0.02-0.12. We are, however, only interested in the *degradation* of these accuracies due to uncertain knowledge of redshifts, and this fact makes our results less dependent on the details of the survey. Finally, for this analysis we assume that the 'mass-observable' (i.e. mass-temperature, or mass-X-ray flux) relation is perfectly known. We have checked that leaving the normalization of this relation as a free parameter to be determined by the data can strongly degrade the fiducial parameter constraints, but it affects the sensitivity to the knowledge of redshifts, which we explore here, much more weakly.

To compute the comoving number of clusters we use the Jenkins et al. (2001) mass function. The required input is the linear power spectrum; for $w = -1$ models, we use the formulae of Eisenstein & Hu (1997) which were fit to the numerical data produced by CMBfast (Seljak and Zaldarriaga 1996). We generalize the formulae to $w \neq -1$ by appropriately modifying the growth function of density perturbations. The total number of objects in any redshift interval centered at z and with width Δz is

$$N(z, \Delta z) = \Omega_{\rm survey} \int_{z-\Delta z/2}^{z+\Delta z/2} n(z, M_{\rm min}(z)) \, \frac{dV(z)}{d\Omega \, dz} \, dz \quad (5)$$

where $\Omega_{\rm survey}$ is the total solid angle covered by the survey, $n(z, M_{\rm min})$ is the comoving density of clusters more massive than $M_{\rm min}$, and $dV/d\Omega dz$ is the comoving volume element.

An incorrect determination of individual cluster redshifts will lead to an incorrect central value of the redshift bin to which these clusters are assigned (see below for the definition of redshift bins). This in turn leads to evaluating the theoretically expected number of clusters $N(z, \Delta z)$ at an incorrect central redshift z, thus biasing the inferred cosmological parameters. Here we represent the uncertainty in the central value of the redshift bin as

$$\sigma_{\rm z,bin} = \sqrt{\frac{\sigma_{\rm clus}^2}{N(z, \Delta z)} + \sigma_{\rm sys}^2}, \quad (6)$$

that is, the redshift error is the sum of the purely statistical (random Gaussian) error per cluster $\sigma_{\rm clus}$ and an irreducible, systematic error $\sigma_{\rm sys}$. The source of the irreducible error could be, for example, a systematic offset in the photometric error determination which affects all clusters in that bin equally. We assume that the irreducible error is uncorrelated between bins. The Fisher matrix is constructed as in Eq. (1), with $O(z) \equiv N(z, \Delta z)$.

[1] http://cosmology.berkeley.edu/group/swlh/acbar/
[2] http://astro.uchicago.edu/sza/
[3] http://bolo.berkeley.edu/apexsz/
[4] http://www.hep.upenn.edu/ angelica/act/index.html
[5] http://astro.uchicago.edu/spt/
[6] http://astro.estec.esa.nl/Planck/

We assume three representative fiducial surveys: 1) South Pole Telescope (SPT), a 4000 sq. deg. survey that will detect clusters through their SZ signature, 2) Planck mission, which we consider to be a 28,000 sq. deg. SZ survey and 3) DUET, a planned X-ray space mission with coverage of 10,000 sq. deg. For the SPT and Planck we use the mass-SZ flux relation from Majumdar & Mohr (2003b), while for DUET we assume the Majumdar-Mohr mass-Xray flux relation. We normalize these analytical 'mass-observable' relations so that all three of these surveys would produce between 18,000 and 25,000 clusters for our fiducial cosmology; we have checked that different normalizations do not change our results dramatically. Nevertheless, the choice of the mass-observable relation is important since $M_{\rm min}$ depends on cosmological parameters and modifies the error budget. To present a range of possibilities, we further consider the SPT survey with fixed limiting masses of 10^{14}, 2×10^{14} and $5\times10^{14}\,h^{-1}M_\odot$; in our fiducial cosmology these three possibilities lead to about 95,000, 20,000 and 1,700 clusters respectively.

Redshifts for the clusters are provided by optical and near-infrared photometric follow-up. These photometric redshifts are calibrated using supplemental spectroscopic observations of a galaxy subset. We assume the Sloan Digital Sky Survey (SDSS) passbands; they have been extensively used for photometric-redshift measurements of moderate-redshift galaxy clusters (Bahcall et al. 2003). The SDSS photometric system (Fukugita et al. 1996) is comprised of five bands: u' peaks at 3500Å with FWHM of 600Å, g' peaks at 4800Å with FWHM of 1400Å, r' peaks at 6250Å with FWHM of 1400Å, i' peaks at 7700Å with FWHM of 1500Å, and z' peaks at 9100Å with FWHM of 1200Å. The redshift bins are defined by where the 4000Å line enters and leaves these bands, and the bin width is typically 0.1-0.2 in redshift. At redshifts greater than about 1.2 the 4000Å line leaves the observable bands and enters the infrared, which makes obtaining the photometric redshifts at higher z much more difficult. To represent the situation in a few years, we assume the redshift bin widths of 0.5 at $z > 1.2$, keeping the error per bin the same; this is roughly equivalent to doubling the error per redshift interval found at $z < 1.2$. We quantitatively discuss the redshift accuracy at high redshift in the following subsection.

4.2. *Results*

The *purely statistical* error in photometric redshifts, $\sigma_{\rm clus}$, is largely irrelevant for cosmological constraints, as can be seen from Eq. (6), since the large number of clusters (in all redshift bins except for those at the highest redshifts) will make error per bin very small. Therefore, the current statistical error with scatter of about $\sigma_{\rm clus} = 0.02$ in redshift contributes negligibly, by itself, to the total error budget, and we assume this statistical error for each individual cluster. However, we find that the results are sensitive to *uncorrelated irreducible systematic* errors in redshift bins, $\sigma_{\rm sys}$. The measurement of photometric redshift primarily relies on the position of the 4000Å break. For each redshift there is a corresponding filter (or overlapping filter pair) which is sensitive at $4000(1+z)$Å. Galaxies at similar redshift and whose redshift determinations rely primarily on the same filters are thus susceptible to common systematic errors. These errors can arise due to improper modeling of the filters, photometric calibration uncertainty, or statistical errors in the redshift calibration process.

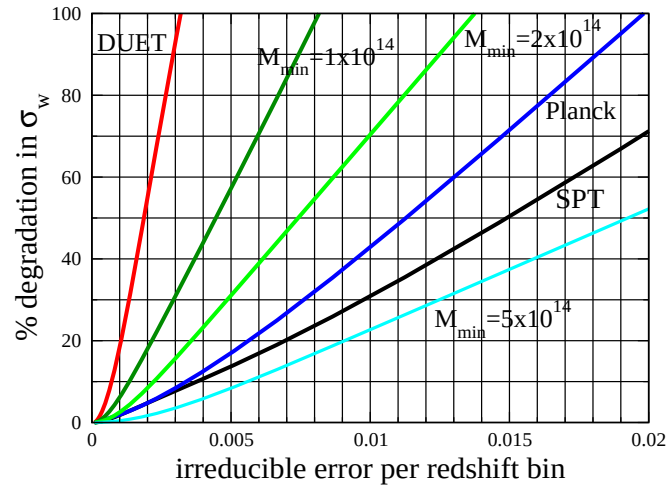

FIG. 4.— Degradation in the measurement accuracy of the equation of state as a function of the *irreducible* redshift error per bin. SPT and Planck curves use the mass-SZ flux relation to compute $M_{\rm min}$, while the DUET curve uses the mass-Xray flux relation. The three other curves show the SPT survey with fixed $M_{\rm min}$ in units of $h^{-1}M_\odot$.

Figure 4 shows the degradation in the accuracy in w as a function of the irreducible error. We show the degradation in w as a representative example, and have checked that degradations in Ω_M and σ_8 are comparable and their range of sensitivities is spanned by the various curves shown in this figure. We see that the X-ray survey is most sensitive to this redshift error; this is not surprising as the X-ray survey runs out of clusters at $z \gtrsim 1.0$ (most of them are actually at $z \lesssim 0.6$) and hence uses a shorter lever arm in redshift than SZ surveys. The differences in various curves show the dependence of the sensitivity of σ_w on details of the survey and, in particular, of the definition of the minimal cluster mass. The irreducible error per bin has to be kept below 0.001-0.005, depending on the survey details, in order for it not to contribute more than $\sim 10\%$ to the error in w and other cosmological parameters. Furthermore, we have explored a range of fiducial surveys, varying the parameter set, sky coverage, and the details of the mass-observable relation, and found that surveys with less power to measure cosmological parameters typically have weaker requirements on the redshift accuracy. While we have shown a range of possibilities in Fig. 4, we note that the exact requirements on the redshift accuracy for any given survey will be known only after the survey in question has started its operation and the accuracy of cluster mass determination from the observed flux or temperature becomes known.

We mentioned earlier that the future accuracy of photometric redshifts at $z \gtrsim 1$ is uncertain. We explore the accuracy in measuring w on redshift information at $z > z_*$ where we let z_* vary between 0.8 and 3.0. Figure 5 shows that not obtaining redshifts for clusters at such redshifts can significantly degrade the performance of SZ surveys (X-ray survey do not have this problem, since they have very few clusters at $z \gtrsim 1$). Redshift information is as-

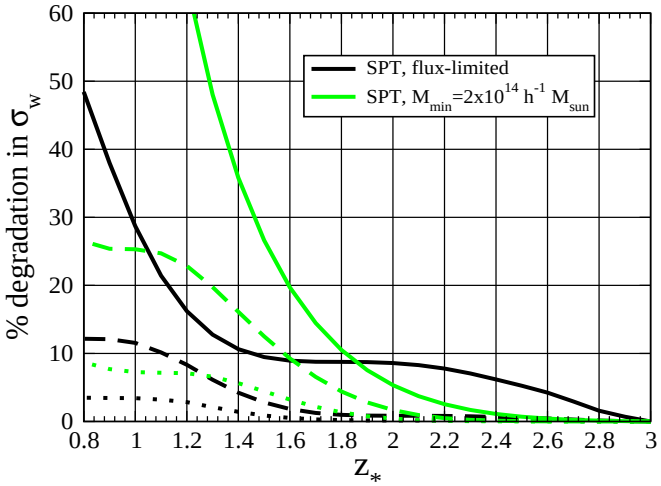

FIG. 5.— Degradation in the measurement accuracy of the equation of state as a function of the redshift z_* beyond which photometric redshift information is poor or nonexistent. Redshift information is assumed to be perfect at $z < z_*$, while at $z > z_*$ we alternatively assume: no redshift information (solid lines), irreducible errors of 0.05 per redshift bin width of $\Delta z = 0.1$ (dashed lines) and 0.02 per redshift bin (dotted lines). We show cases for the flux-limited SPT and for SPT with fixed $M_{\rm min}$. Note that an X-ray survey would show no degradation in σ_w at all, since essentially all of its clusters are at redshifts $\lesssim 1$.

sumed to be perfect at $z < z_*$, while at $z > z_*$ we alternatively assume no redshift information (solid lines), irreducible errors of 0.05 per redshift bin width of $\Delta z = 0.1$ (dashed lines) and 0.02 per redshift bin (dotted lines). We show cases for the flux-limited SPT and for the same experiment with fixed $M_{\rm min}$. This figure shows that, while missing information at $z \gtrsim 2$ can be tolerated, having some redshift information in the region $1 \lesssim z \lesssim 2$ is very important. While photometric redshifts in this intermediate interval are difficult to obtain because of the lack of prominent features, we see that relatively modest information (redshift bias accurate to about 0.05 *per bin*) is sufficient to recover most of the information obtainable with perfect-redshift accuracy. We also checked that the error degradations in Ω_M and σ_8 are very similar to that in w.

Finally, we note that our cluster count requirements were based on the degradation in the accuracy of measuring equation of state w, which, in our fiducial surveys, is measured to accuracy $\sigma_w = 0.02 - 0.12$. Weaker constraints on w, due to the inclusion of systematics, new parameters (such as the mass-observable normalization and slope), or else due to smaller sky coverage of the survey, will lead to *weaker* redshift requirements. Nevertheless, precision measurement of dark energy parameters is one of the principal goals of future number-count surveys, and it is expected that they will be powerful enough to complement concurrent supernova surveys. Therefore, redshift control less stringent than that advocated here would weaken the power of number count surveys to probe dark energy.

5. CONCLUSIONS

We considered how inexact redshifts affect future SNe Ia and number-count surveys. We treated the redshifts as additional parameters whom we assigned priors equal to their assumed measurement accuracy. Requiring that the redshift uncertainty do not contribute more than $\sim 10\%$ to the error budget in cosmological parameters, we imposed requirements on the redshift accuracy.

For a future survey that studies ~ 3000 supernovae out to $z = 1.7$ (e.g. the SNAP space telescope) we find that, with accurate redshift measurements of $dz \lesssim 0.001$ for $z < 0.1$ supernovae, fairly poor redshift measurements can be tolerated at higher redshifts. Without this accurate measurement at low redshift, however, a fairly precise redshift measurement of $dz \lesssim 0.002$ would be required over the full redshift range. Photometric redshifts are probably not an option, since spectral information is necessary to identify the SN type and control a variety of systematic errors. Spectroscopy can be provided using subpixel interpolation of galaxy data from an on-board low-dispersion $R \sim 100$ spectrograph (which is designed to measure broad supernova features). Supplemental high-resolution ground-based observations using 10m-class telescopes, adaptive optics, and OH suppression can provide precise redshifts as necessary and to cross-check the redshifts from the low-dispersion spectrograph. We thus conclude that redshift uncertainty will not significantly contribute to the error budget in the accurate measurement of dark-energy parameters that SNAP can deliver.

For future wide-field cluster count surveys, such as SPT, Planck or DUET, we find that the purely statistical errors are largely irrelevant as long as they are reasonably small (error of $\lesssim 0.02$ per cluster) because they will average out due to the large number of clusters around any given redshift. However, the irreducible, systematic error that doesn't decrease with increasing number of clusters drives the redshift requirements. This irreducible redshift-independent error has to be kept below 0.001-0.005 per redshift bin. The widths of the redshift bins are determined by how the redshift signature (say, the 4000Å break line) goes through the filter set of the redshift follow-up experiment, and here for illustration we assumed filters from the Sloan Digital Sky Survey. We found that the typical required redshift accuracy is more stringent for X-ray surveys since they have few clusters at $z \gtrsim 1$ and therefore use a shorter lever arm in redshift. SZ surveys benefit from their longer lever arm, but, of course, only if their high-redshift clusters have decent redshift information. Obtaining redshifts for high-redshift clusters, therefore, should be an important goal of any redshift follow-up survey. While the photometric accuracy at redshifts greater than unity is highly uncertain at present, our analysis indicates that the lack of redshift information at $z \gtrsim 2$ does not significantly degrade the cosmological constraints, while at redshifts $1 \lesssim z \lesssim 2$ crude photometric information is sufficient to assure small degradation in constraints on w (see Fig. 5). With the current rate of progress in photometric redshift techniques, this should be a feasible goal within the next few years.

We thank Jim Bartlett, Josh Frieman, Adrian Lee, and Tim McKay for useful discussions. We particularly thank Eric Linder for pointing out the importance of accurate redshifts for low-z SNe, and Jim Annis and Martin White for comments on an early draft of the paper. DH and LMK are supported by the DOE grant to CWRU. AK was supported by the Director, Office of Science, of the U.S.

Department of Energy under Contract No. DE-AC03-76SF00098.

REFERENCES

Akerlof, C. et al. 2004, in preparation
Aldering, G. et al. 2002, Proc. SPIE, 4836, 61
Bartlett, J. 1999, astro-ph/0001267
Battye, R.A. & Weller, J. 2003, Phys. Rev. D, 68, 083506
Bahcall N.A. et al. 2003, ApJS, 148, 243
Benson, A.J., Reichardt, C. & Kamionkowski, M. 2002, MNRAS, 331, 71
Cardelli, J.A., Clayton, G.C., & Mathis, J.S. 1989, ApJ, 345, 245
Collister, A.A. & Lahav, O. 2003, MNRAS, submitted (astro-ph/0311058)
Csabai, I. et al. 2003, AJ, 125, 580
Eisenstein, D. & Hu, W. 1997, ApJ, 511, 5
Fernández-Soto, A., Lanzetta, K.M., Chen, W.-W., Levine, B. & Yahata,N. 2002, MNRAS, 220, 889
Frieman, J.A., Huterer, D., Linder, E. & Turner, M.S. 2003, Phys. Rev. D, 67, 083505
Fukugita, M. et al. 1996, AJ, 111, 1748
Haiman, Z., Mohr, J.J., & Holder, G.P. 2001, ApJ, 533, 545
Holder, G.P. & Carlstrom, J. 2001, ApJ, 558, 515
Holder, G.P., Haiman, Z., & Mohr, J.J. 2001, ApJ, 560, L111
Hu, W. 2003, Phys. Rev. D, 67, 081304
Hu, W., & Kravtsov, A. 2003, Astrophys. J., 584, 702
Huterer, D. & Turner, M.S. 2001, Phys. Rev. D, 64, 123527
Huterer, D. & White, M. 2002, ApJ, 578, L95
Jenkins, A. R. et al. 2001, MNRAS, 321, 372
Kim, A., Linder, E., Miquel, R., & Mostek, N. 2004, MNRAS, 347, 909
Kneissl, R. et al. 2001, MNRAS, 328, 783
Knop, R.A. et al. 2003, ApJ, submitted (astro-ph/0308368)
Krauss, L.M. & Turner, M.S. 1995, Gen. Rel. Grav., 27, 1137
Levine, E.S., Schulz, A.E., & White, M. 2002, ApJ, 577, 569
Lima, M. & Hu, W. 2004, Phys. Rev. D, submitted (astro-ph/0401559)
Majumdar, S., & Mohr, J.J. 2003a, ApJ, 585, 603
Majumdar, S., & Mohr, J.J. 2003b, ApJ, submitted (astro-ph/0305341)
Molnar, S.M., Haiman, Z., Birkinshaw, M. & Mushotzky, R.F. 2003, ApJ, submitted (astro-ph/0309807)
Nugent, P., Kim, A., & Perlmutter, S. 2002, PASP, 114, 803
Ostriker, J.P. & Steinhardt, P.J. 1995, Nature, 377, 600
Perlmutter, S. et al. 1997, ApJ, 483, 565
Perlmutter, S. et al. 1999, ApJ, 517, 565
Pierre, M. et al. 2003, A&A, submitted (astro-ph/0305191)
Romer, A.K., Viana, P.T.P., Liddle, A.R. & Mann, R.G. 2001, ApJ, 547, 594
Seljak, U. & Zaldarriaga, M. 1996, ApJ 469, 437
Spergel, D.N. et al. 2003, ApJ, 148, S175
Tegmark, M. et al. 2003, Phys. Rev. D, submitted (astro-ph/0310723)
Tonry, J.L. et al. 2003, ApJ, 594, 1
Vanzella, E. et al. 2003, A&A, submitted (astro-ph/0312064)
Weller, J., Battye, R. & Kneissl, R. 2002, Phys. Rev. Lett., 88, 231301
White, M. 2003, ApJ, 597, 650
White, M., Hernquist, L. & Springel, V 2002, ApJ, 579, 16
White, M., van Waerbeke, L & Mackey, J. 2002, ApJ, 575, 640